Interior Point Methods Applied to Basis Pursuit

Paula Aparecida Kikuchi
(IMECC – Unicamp)

Prof. Daniela Renata Cantane
(Unesp – Botucatu)
Prof. Aurelio Ribeiro Leite de Oliveira
(IMECC – Unicamp)

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Outline

Motivation

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Given the signal $x \in \mathbb{R}^N$ and a orthonormal bases $\{\psi_i\}_{i=1}^N$, we can express x as:

$$x = \sum_{i=1}^{N} \alpha_i \psi_i \text{ ou } x = \Psi \alpha \tag{1}$$

where $\alpha_i = \langle x, \psi_i \rangle$.

Motivation

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Compressible signals are well approximated by $K-sparse$ representations

Transform Coding

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- The K largest coefficients are located and the $N - k$ smallest coefficients are discarded.

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- The locations of the large coefficients must be encoded, thus introducing an overhead.

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- We can write:

$$s = \Phi x = \Phi \Psi \alpha = \Theta \alpha$$

where $\Theta = \Phi\Psi$ is an $M \times N$ matrix.

Compressive Sensing

The problem consists in:

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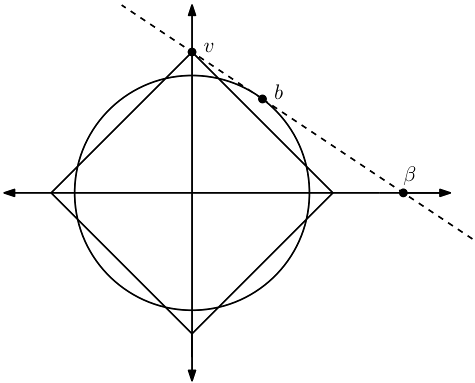
- A reconstruction algorithm to recover x from only M measurements s.

Geometric Motivation

Consider the search for α signal having representation sparser and respects the linear equation that restricts its position in $\mathbb{R}^2$ on the dotted line.



Basis Pursuit

Our goal is to apply Interior Point Methods to the problem:

$$\begin{aligned} \text{minimize} \quad & \|\alpha\|_1 \\ \text{subject to} \quad & \Theta\alpha = s \end{aligned} \tag{2}$$

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In search of the sparse representation, applies *Basis Pursuit*.

To solve the problem *Basis Pursuit*, Chen implements the Primal-Dual Logarithmic Barrier Method.

Our goal will be get more efficient results, implementing Interior Point Methods to the problem in question.

Signal Representation

Definition

(Dictionary) A dictionary is a collection of parameterized waveforms $D = (\phi_\gamma : \gamma \in \Gamma)$*, with waveforms* ϕ_γ *discrete-time signals called atoms, which may also be viewed as a vector in* $\mathbb{R}^n$*.*

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Dictionaries are defined as complets when they contain exactly n atoms, *overcomplete* when contain more than n atoms, or *undercomplete* when contain fewer than n atoms.

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Dictionaries examples: stationary wavelets, wavelet packets, cosine packets, chirples, warplets, Gabor dictionaries, wavelets and steerable wavelets.

Signal Representation

Let s be a discrete-time digital signal or a vector in $\mathbb{R}^n$, we assume a decomposition of the signal as

$$s = \sum_{\gamma \in \Gamma} \alpha_\gamma \phi_\gamma. \tag{3}$$

Consider that we have a discrete dictionary with p *waveforms* and a Φ matrix whose columns correspond to p *waveforms* ($\Phi : n \times p$), we can rewrite (3) as:

$$\Phi\alpha = s, \tag{4}$$

wherein $\alpha = (\alpha_\gamma)$ is the vector of coefficients in (3).

Signal Representation

There are several methods proposed to represent signals with *overcomplete* dictionaries. Some examples are:

- Method of Frames.
- Matching Pursuit.
- The Best Orthogonal Basis,
- Basis Pursuit.

Signal Representation

Basis Pursuit

The principle of Basis Pursuit is to find a representation of the signal whose coefficients have minimal norm 1. Formally, one solves the problem:

$$\begin{array}{ll} \text{minimize} & \|\alpha\|_1 \\ \text{subject to} & \Phi\alpha = s \end{array} \tag{5}$$

Basis Pursuit

Although the Basis Pursuit problem (5) involves nonlinear optimization, it can be equivalently reformulated as a linear program in the standard form:

$$\begin{aligned} \text{minimize} \quad & c^T x \\ \text{subject to} \quad & Ax = b, \\ & x \geq 0. \end{aligned} \tag{6}$$

Basis Pursuit

Consider α as:

$$\alpha = u - v, \text{ with } u \text{ and } v \text{ not negatives}$$

and with $\|\alpha\|_1$ upper bound of $e^T u + e^T v$, follows

$$\Phi\alpha = \Phi\left(u - v\right)$$
and
$$e^T u + e^T v = \|\alpha\|_1$$

Basis Pursuit

We obtain the following equivalent problem:

$$\begin{array}{rl} \text{minimize} & e^T u + e^T v \\ \text{subject to} & \Phi\,(u - v) = s, \\ & (u, v) \geq 0, \end{array}$$

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We obtain the linear problem in standard form.

Formulation

We will use the formulation proposed by Gill, Murray and Saunders, rewriting the problem as the following perturbed linear program:

$$\begin{array}{ll} \text{minimize} & c^T x + \frac{1}{2}\|\gamma x\|_2^2 + \frac{1}{2}\|p\|_2^2 \\ \text{subject to} & Ax + \delta p = b, \\ & x \geq 0, \end{array} \quad (7)$$

where γ and δ are small $\left(10^{-4}\right)$ perturbation parameters;

Logarithmic Barrier

One can associate the perturbed linear problem (7) with a logarithmic barrier subproblem:

$$\begin{aligned} \text{minimize} \quad & c^T x + \frac{1}{2}\|\gamma x\|_2^2 + \frac{1}{2}\|p\|_2^2 - \mu \sum_{i=1}^{m} \ln(x_i) \\ \text{subject to} \quad & Ax + \delta p = b. \end{aligned}$$

where the constraint $x \geq 0$ is implicit. As $\mu \to 0$ the solution of the logarithmic barrier problem converges to the solution of the perturbed linear problem.

First Order Necessary Conditions

Necessary first order conditions of the problem of minimizing the Lagrangian function:

$$\text{minimize } c^T x + \frac{1}{2}\|\gamma x\|_2^2 + \frac{1}{2}\|p\|_2^2 - \mu \sum_{i=1}^{m} \ln(x_i) + y^T (b - Ax - \delta p)$$

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\nabla_p L &= \delta y - p = 0 \Rightarrow p = \delta y \\
\nabla_y L &= Ax + \delta p - b = Ax + \delta^2 y - b = 0
\end{aligned}$$

First Order Necessary Conditions

And complementarity condition for the dual variable z we have:

$$ZXe - \mu e = 0$$

Newton's Method

The Newton search directions $(\Delta x, \Delta y, \Delta z)$ satisfy:

$$\left(ADA^T + \delta^2 I\right) \Delta y = r - AD\left(X^{-1}v - t\right) \tag{8}$$

$$\Delta x = DA^T \Delta y + D\left(X^{-1}v - t\right) \tag{9}$$

$$\Delta z = X^{-1}v - X^{-1}Z\Delta x \tag{10}$$

where $D = \left(X^{-1}Z + \gamma^2 I\right)^{-1}$.

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- Each barrier iteration consists of a Newton step adjustment followed by a decrement in the barrier parameter μ. Decrease μ monotonically and more rapidly if larger steps are taken:

$$\mu \leftarrow (1 - min\,(\rho_p, \rho_d, 0, 99))\,\mu$$

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- The method converges when $\left\| b - Ax - \delta^2 y \right\|_2$, $\left\| c + \gamma^2 x - z - A^T y \right\|_2$ e $z^T x$ are small enough.

BP_Interior

The problem BP_Interior, implemented by Chen and contained in *Atomizer*, solves the problem *Basis Pursuit* using the same procedure described above.

$$\alpha \Longleftrightarrow u - v$$

$$x \Longleftrightarrow (u; v)$$

$$c \Longleftrightarrow (e; e)$$

$$b \Longleftrightarrow s$$

$$A \Longleftrightarrow (\Phi, -\Phi)$$

$$m \Longleftrightarrow 2p$$

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- $$\mu_0 = 0,01 \frac{\|f\|_2}{\sqrt{2n}}$$

  where f is the vector whose entries are given by the optimality conditions:

  $$\left\|b - Ax - \delta^2 y\right\|_2,$$

  $$\left\|c + \gamma^2 x - z - A^T y\right\|_2,$$

  $$z^T x.$$

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Is projected $dx = x - x_0$ in the null space of Φ by Conjugate Gradient in order to ensure feasibility.

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- Superresolution: We should obtain a resolution of sparse objects that has much higher resolution than that possible with traditional non-adaptive approaches;
- Stability: Small perturbations of signal should not seriously degrade the results;
- Speed.

Development of Proposed Methods

Therefore, we modify the Primal-Dual Logarithmic Barrier Method, in the program BP_Interior, to obtain better efficiency. We apply Predictor-Corrector Primal-Dual Logarithmic Barrier Method to problem BP.

Predictor-Corrector Primal-Dual Logarithmic Barrier Method

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- **Centering Direction**, prevents the solutions are close to the edges of the polytope defined by the constraints along the iterations.
- **Nonlinear Correction Direction**, calculate the non-linear correction, trying to compensate the linear approximation of Newton's Method.

Predictor-Corrector Primal-Dual Logarithmic Barrier Method

Note that the nonlinear term in BP_Interior, corresponds complementarity condition, noting that we define $v = \mu e - Zx$ in BP_Interior.
The two steps, corresponding both times we apply Newton's Method can be summarized in:

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The two steps, corresponding both times we apply Newton's Method can be summarized in:

- In the first step we consider v such that $v = -Zx$,
- In the second step $v = \mu e - Zx - \Delta Z \Delta X e$, where Δz e Δx were obtained in the first step

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$$F(x,y,z,p) = \begin{pmatrix} A^T y - \gamma^2 x - c + z \\ \delta y - p \\ Ax + \delta p - b \\ ZXe - \mu e \end{pmatrix} = \mathbf{0},$$

where z is a dual vector, X and Z are the diagonal matrices formed by the elements of the vectors x and z, respectively.

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$$\Delta p = \delta \Delta y - r,$$

with $D = \left(X^{-1}Z + \gamma^2 I\right)^{-1}$.

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Adding affine scaling direction , centering direction, and the correction direction given by the Predictor-Corrector, let us consider in the first step:

- $w = -Zx$

  and in the second step:

- $w = \mu - Zx - \Delta Z \Delta X e$.

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- To choose the step length (ρ_p, ρ_d), we will do just as determined to Predictor-Corrector Primal-Dual Logarithmic Barrier Method, where we always choose the largest possible, provided that the point $x^{(k+1)}$ and $z^{(k+1)}$ are interior points.

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- Every Newton step we grow barrier parameter μ monotonically and faster if large steps are taken:

$$\mu \leftarrow \left(1 - min\left(\rho_p, \rho_d, 0,99\right)\right)\mu.$$

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Computational Tests

Computational Tests

The following are the results for signs:

- *TwinSine-1*,
- *WernerSorrows*,
- *Carbon*,
- *TwinSine-2*,
- *FM-Cosine*,
- *Gong*,
- *Dynamic-0*,
- *Dynamic-2* and
- *MultiGong*.

Computational Tests

Computational Tests

For this representation we apply the BP_Interior Method of Chen, and our modifying methods of BP_Interior:

- Predictor-Corrector Primal-Dual Logarithmic Barrier Method - BP_InteriorPC,
- and our variant of Predictor-Corrector Primal-Dual Logarithmic Barrier Method - BP_InteriorPC1.

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The numerical experiments were implemented in Matlab R2010a, operating system Ubuntu 11.04, processor Intel ® core i7 2600, 3.4 Ghz, 4 GB memory DDR3, RAM clock 1333Mhz.

Problem Data

Signal	Size of Problem	Dictionary	par1	par2	par3
TwinSine-1	256	DCT	4	0	0
WernerSorrows	1024	CP	6	seno	0
Carbon	1024	WP	10	qmf	0
TwinSine-2	256	DCT	4	0	0
FM-Cosine	1024	CP	6	seno	0
Gong	1024	CP	10	seno	0
Dynamic-0	256	DCT and DIRAC	MekeList(4,0)	0	0
Dynamic-2	256	DCT and DIRAC	MekeList(4,0)	0	0
MultiGong	256	MDC	8	1	0

Computational results for the objective function and processing time

Signal	BP_Interior	
	OF	**Time**
TwinSine-1	2,00933e+00	0,1
WernerSorrows	5,07482e+02	249,2
Carbon	6,00247e+00	19,8
TwinSine-2	2,01150e+00	0,1
FM-Cosine	2,52872e+02	237,3
Gong	4,73171e+00	1924,5
Dynamic-0	6,01964e+00	0,3
Dynamic-2	4,03672e+02	0,4
MultiGong	2,43810e+01	5,6

where OF corresponds to the value of the objective function.

Computational results for the objective function and processing time

Signal	BP_InteriorPC		BP_InteriorPC1	
	OF	Time	OF	Time
TwinSine-1	2,00934e+00	0,1	2,00934e+00	0,1
WernerSorrows	5,07587e+02	130,0	5,07576e+02	124,1
Carbon	6,00003e+00	19,6	6,00003e+00	20,2
TwinSine-2	2,01108e+00	0,1	2,01108e+00	0,1
FM-Cosine	2,52885e+02	210,9	2,52896e+02	208,5
Gong	4,73088e+00	3501,3	4,72132e+00	11587,9
Dynamic-0	6,01902e+00	0,4	6,01907e+00	0,5
Dynamic-2	4,02187e+02	0,7	4,02187e+02	0,7
MultiGong	2,44009e+01	7,2	2,43895e+01	7,3

where OF corresponds to the value of the objective function.

Tests

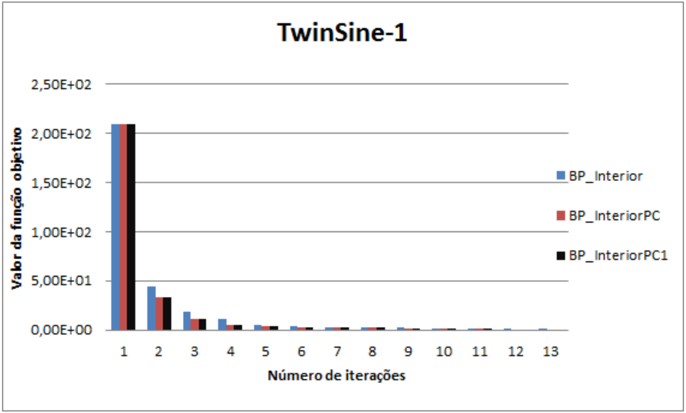
- BP_Interior, BP_InteriorPC and BP_InteriorPC1 have equivalent computational performance relative to the value of the objective function.

Tests

The convergence time for BP_Interior, BP_InteriorPC and BP_InteriorPC1 were similar, with the most significant differences obtained for:

- The signal *WernerSorrows*, where
  BP_Interior get $249, 2$ seconds for the convergence,
  BP_InteriorPC, $130, 0$ seconds and
  BP_InteriorPC1, $124, 1$ seconds;
- and signal *Gong*, we get very different times for the three methods,
  BP_Interior get $1924, 5$ seconds for the convergence,
  BP_InteriorPC $3501, 3$ seconds and
  BP_InteriorPC1 $11587, 9$ seconds.

Tests


TwinSine-1
Valor da função objetivo
2,50E+02
2,00E+02
1,50E+02
1,00E+02
5,00E+01
0,00E+00
1
2
3
4
5
6
7
8
9
10
11
12
13
Número de iterações
BP_Interior
BP_InteriorPC
BP_InteriorPC1

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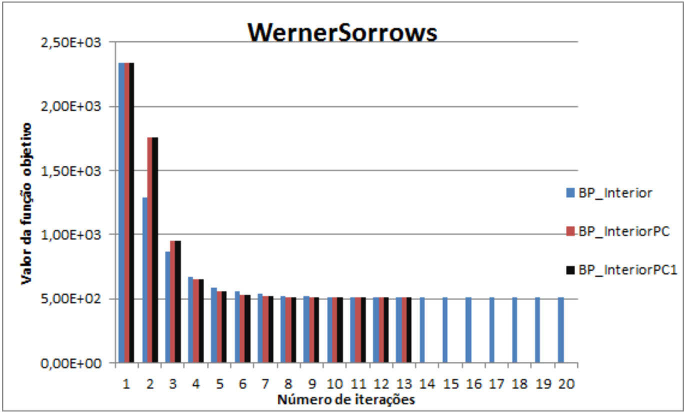
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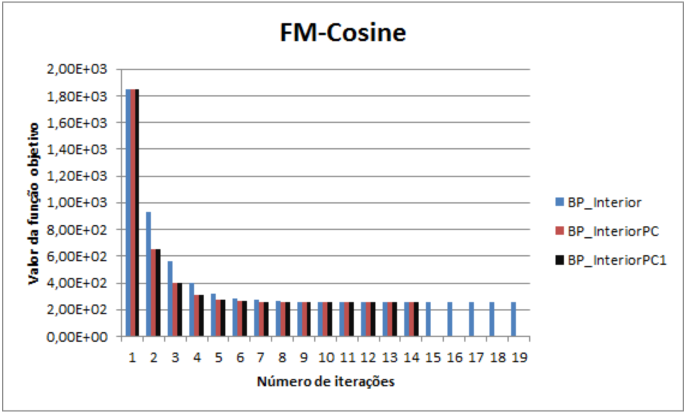
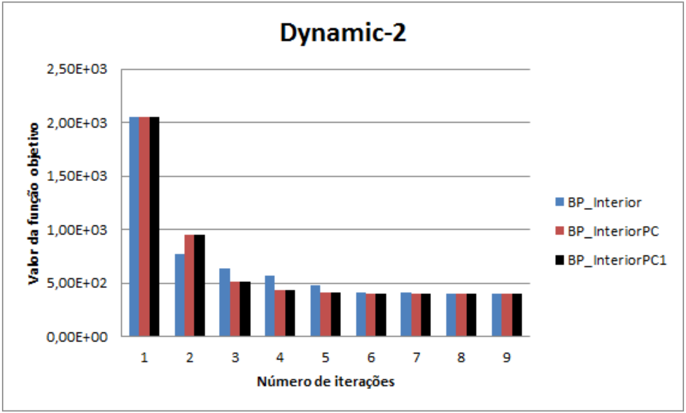
Valor da função objetivo
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Número de iterações
BP_Interior
BP_InteriorPC
BP_InteriorPC1

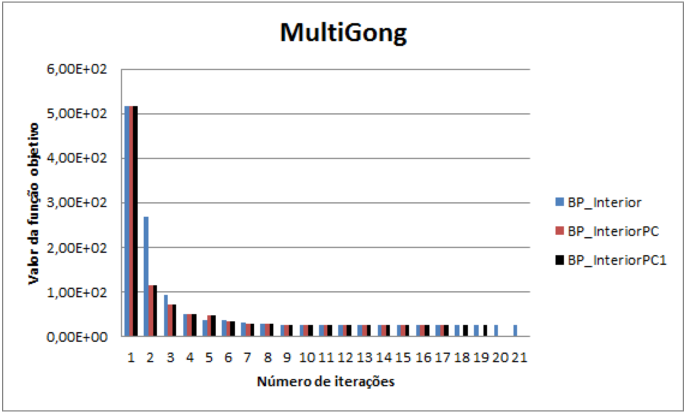
Tests


FM-Cosine
2,00E+03
1,80E+03
1,60E+03
1,40E+03
1,20E+03
1,00E+03
8,00E+02
6,00E+02
4,00E+02
2,00E+02
0,00E+00
Valor da função objetivo
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19
Número de iterações
BP_Interior
BP_InteriorPC
BP_InteriorPC1

Tests


Dynamic-2
2,50E+03
2,00E+03
1,50E+03
1,00E+03
5,00E+02
0,00E+00
Valor da função objetivo
1
2
3
4
5
6
7
8
9
Número de iterações
BP_Interior
BP_InteriorPC
BP_InteriorPC1

Tests


MultiGong
6,00E+02
5,00E+02
4,00E+02
3,00E+02
2,00E+02
1,00E+02
0,00E+00
Valor da função objetivo
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
Número de iterações
BP_Interior
BP_InteriorPC
BP_InteriorPC1

Number of iterations of the program and the method of conjugate gradient.

Sinal	BP_Interior		BP_InteriorPC		BP_InteriorPC1	
	It	ItCG	It	ItCG	It	ItCG
TwinSine-1	11	59	9	90	9	90
WernerSorrows	18	20883	11	10850	11	10520
Carbon	8	85	6	99	6	99
TwinSine-2	9	43	9	97	9	98
FM-Cosine	17	20098	12	17891	12	18026
Gong	21	11510	19	21681	23	71812
Dynamic-0	7	40	5	54	5	54
Dynamic-2	7	52	7	91	7	91
MultiGong	19	853	15	1097	17	1457

Tests

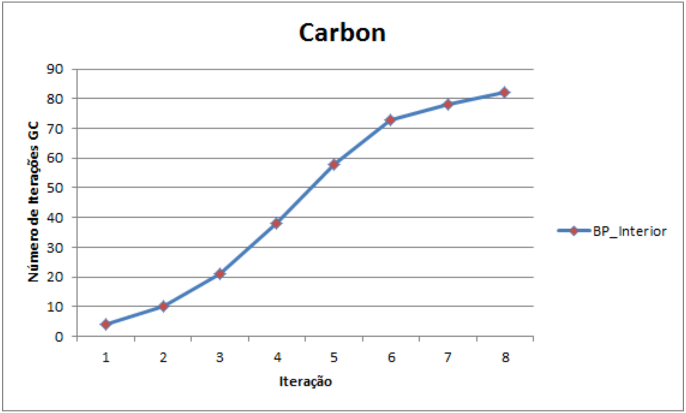
- When the number of iterations performed by BP_InteriorPC was not less than the number get by BP_Interior, this demonstrated to be equal.
- BP_InteriorPC1 got more iterations that BP_Interior only for the signal *Gong*, with a difference of only two iterations.

Tests

In relation to Conjugate Gradient Method, we note that the number of iterations increases constantly. This happens because:

- The initial solution is near the center of the feasible region, so in initial iteractions the system of equations is well conditioned and the method converges quickly.
- As $x^T z$ converges to 0, z/x converges to infinity or 0, thus the matrix $D = \left(X^{-1}Z + \gamma^2 I\right)$ e a matriz $\left(ADA^T + \delta^2 I\right)$ becomes ill-conditioned, and the method takes to converge.

Tests


Carbon
Número de Iterações GC
90
80
70
60
50
40
30
20
10
0
1
2
3
4
5
6
7
8
Iteração
BP_Interior

Conclusions

- Obtain better perfomance with affine scaling direction , centering direction, and the correction direction of the Predictor-Corrector Method.
- Although BP_InteriorPC1 has obtained a very similar result to BP_InteriorPC, BP_InteriorPC achieved the best results, therefore the most efficient.

Futures Prospects

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- In order to obtain better results, precondition the matrix of the linear system, to obtain a smaller number of iterations in Conjugate Gradient Method.

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- In order to obtain better results, precondition the matrix of the linear system, to obtain a smaller number of iterations in Conjugate Gradient Method.
- Check the application of methods for larger dictionaries, considering if this is feasible in real applications.

Acknowledgement

CAPES - Coordenação de Aperfeiçoamento de Pessoal de Nível Superior

Thank you!